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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 18016.1-EL-A	2. GOVT ACCESSION NO. AD-A119 011 N/A	3. RECIPIENT'S CATALOG NUMBER N/A
4. TITLE (and Subtitle) Three-Dimensional Object Recognition Using N-Dimensional Chain Codes		5. TYPE OF REPORT & PERIOD COVERED Reprint
		6. PERFORMING ORG. REPORT NUMBER N/A
7. AUTHOR(s) Frank P. Kuhl, Didier J. Charpentier, Charles R. Giardina Fairleigh, O. Robert Mitchell		8. CONTRACT OR GRANT NUMBER(s) DAAG29 81 K 0088
9. PERFORMING ORGANIZATION NAME AND ADDRESS Purdue University, West Lafayette, IN		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS N/A
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office, P. O. Box 12211, Research Triangle Park, NC 27709		12. REPORT DATE 1982
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Submitted for announcement only.

Published in: Image Processing and Pattern Recognition Southcon/82; Electronic Show & Convention; Mar 23-25; Orlando, FL.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

DTIC ELECTE SEP 8 1982 S D B

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

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THREE-DIMENSIONAL OBJECT RECOGNITION USING N-DIMENSIONAL CHAIN CODES

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ABSTRACT

Recognition of class and aspect angle of a rigid three-dimensional object in free space is facilitated by using N-dimensional chain codes in feature space. In both radar and image recognition systems features have previously been described that are invariant to object rotation about the observer's line of sight to the object center of gravity. However, rotations about the remaining two orthogonal axes produce changes of the features and a full description of the object becomes a toroidal surface in N-dimensional feature space. N-dimensional chain codes are introduced in this paper to give a line approximation of such a surface that is easily used to identify an unknown object and to estimate its aspect angle.

1. INTRODUCTION

N-dimensional chain codes are a direct extension of the familiar and extremely useful concept of two-dimensional chain codes, which are used to make piecewise-linear approximations of continuous curves with a small set of directed line segments. In this paper straight-forward procedures are given for generating a chain code in N-dimensions, either from a continuous curve or from sample points of the curve. A harmonic description of a closed chain code is derived as well as a bound on the error introduced by truncating the harmonic content. Comparisons are made of the information required to encode a curve with a chain code versus conventional full-coordinate description of approximation points on the curve. N-dimensional codes can be used for efficient storage of toroidal classifications in an N-dimensional feature space and to facilitate comparison of known classifications with the features of an unknown object. An application of N-dimensional codes is shown to the successful recognition of class and aspect angle of aircraft.

2. CHAIN ENCODING OF 2-DIMENSIONAL ARBITRARY GEOMETRIC CURVES

A method of angular quantization and digital encoding of arbitrary 2-dimensional geometric curves has been proposed by Freeman and is extended here for N-dimensional curves. The method for 2-dimensional curves consists of superimposing a square grid on the curve, and approximating the curve with those grid-intersection points, in the order of a trace progressing along the curve, which lie closest to the curve. An example of a curve approximated in this manner is shown in Figure 1. A trace starting from point X is moved along the curve until a grid line is intersected at point B. The distances AB and CB are compared, and since AB is less than CB, the grid intersection point A is chosen as an approxi-

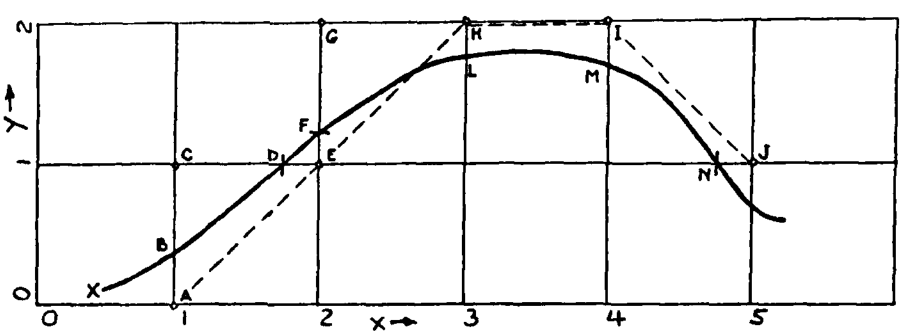

Fig. 1. Approximation of a curve by closest grid intersection points.

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mation point. The trace then proceeds to the next grid intersection at point D. The distance ED is less than CD; therefore point E is chosen as an approximation point. The trace intersects a grid line again at point F, and EF is less than GF, but point E has already been designated as an approximation point and cannot be used twice in succession. The approximation points H, I, J are found in a similar manner as the trace progresses along the curve and intersects grid lines at points L, M, N, respectively. If AB = CB, for example, a consistent choice of approximation points is obtained by using the candidate point closest to the origin of the coordinate axes. Since the curve is assumed to be continuous, it can progress from an approximation point to only one of eight possible neighboring points on the grid. Each possible progression between a pair of approximating points is called a link or vector, and the curve is described by the succession of these vectors. For example, the vector chain for the curve in Figure 1 is shown by the sequence of four straight dotted lines between points A, E, H, I, J.

Each vector encodes an incremental change in position, length, and directional angle of the trace as it progresses along the curve. It is proposed here that the chain code consist of the sequential listing of the incremental changes of position $(\Delta x, \Delta y)$ for each vector; e.g., the link chain V_1 in Figure 1 is as follows:

$$V_1 = (1,1)(1,1)(1,0)(1,-1)$$

The incremental change in length due to a particular vector $(\Delta x, \Delta y)$ is $\Delta t = (\Delta x^2 + \Delta y^2)^{1/2}$ and the directional cosines of the vector for the x and y dimensions are, $\Delta x/\Delta t$ and $\Delta y/\Delta t$, respectively. Note that Δx and Δy can take on only the values -1, 0, and 1.

3. CHAIN ENCODING OF N-DIMENSIONAL ARBITRARY GEOMETRIC CURVES

A continuous arbitrary N-dimensional curve can be chain encoded by mapping to intersection points of an N-dimensional grid. Approximation points are generated in a fashion completely analagous to the 2-dimensional case when the curves crosses a quantization interval. The crossing point is mapped to the nearest grid intersection and a vector V_i is generated, consisting of the incremental variation in each dimension between the approximation points P_{i-1} and P_i. The vector (V) is expressed as

$$V = \mathop{T}_{i=1}^{M^*} v_i \tag{1}$$

where v_i has N components $(X_{i1}, X_{i2}, \ldots, X_{iN})$ corresponding to the number of dimensions in the space. A 3-dimensional example of vector labeling and encoding is shown in Figure 2. Note that since the maximum variation between approximation points is one grid unit in each dimension a vector component X_{ij} can only take on the values of -1, 0, and +1. A vector train in N dimensions may be projected into a lower L-dimensional space by omitting the components of the original vector in the N-L dimensions not of interest and eliminating any resulting null vectors. In Figure 2, the projection of the vector train

$$V_{123} = (1,0,-1)(0,1,0)(0,1,0)(0,1,-1)(1,0,0)$$
$$(1,-1,0)(0,-1,1)(1,0,0)$$
$$(1,-1)(0,-1)(1,0)(1,0)(0,1)(1,0),$$

where two null vectors have been removed.

The length Δt_i of the vector v_i is

$$\Delta t_i = \Big(\sum_{j=1}^{N} |X_{ij}|\Big)^{1/2} \tag{2}$$

and the length t_q of the first q links in the chain is

$$t_q = \sum_{i=1}^{q} \Delta t_i \tag{3}$$

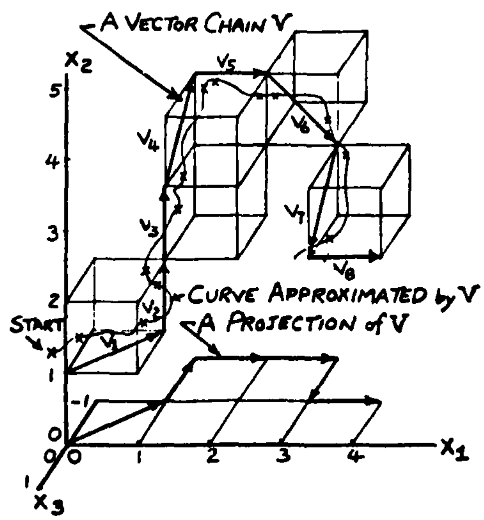

Fig. 2. A 3-D vector chain approximation of a curve and its projection on the x_1-x_3 plane.

For a closed vector consisting of K vectors the period T of the chain is

$$T = t_K \tag{4}$$

4. INFORMATION CONTENT OF THE CHAIN CODE

For N dimensions there are 3^N possible vector types, including the null vector, requiring a minimum of $N \log_2 3$ binary bits of computer-storage per vector. The corresponding bit storage requirement for a single approximation point is $N \log_2 S$, where S is the value of the maximum coordinate in the space. The ratio of total bit storage requirements for recording the full coordinate information for K approximation points along a continuous curve versus vector encoding for the same K approximation points is

$$\frac{K\ N \log_2 S}{(K-1)\ N \log_2 3 + N \log_2 S} \tag{5}$$

For large K, the ratio approaches $\log_3 S$ and vor values of $S > 3$ the vector encoding provides significant economy in storage requirements.

The minimum bit requirement for a vector can be approached only by using special coding schemes. The particular cases for N = 2 and N = 3 have been treated by Freeman[1] and Ruttenburg[2]. For $N \geq 4$, since each component can take on 3 values, and because of the binary nature of digital computers, 2N bits per vector are often used and the storage ratio will approach $.5 \log_2 S = .793 \log_3 S$ for large N. For this more general case, storage economy is realized when $S > 4$.

5. HARMONIC DESCRIPTION OF A CLOSED N-DIMENSIONAL CHAIN CODE

The Fourier Series expansion of the projection of a closed N-dimensional chain code on the x_j axis is defined as

$$x_j(t) = A_{j0} + \sum_{n=1}^{\infty} a_{jn} \cos \frac{2n\pi t}{T} + b_{jn} \sin \frac{2n\pi t}{T} \tag{6}$$

where,

$$A_{j0} = \frac{1}{T}\int_0^T x_j(t)\, dt$$

$$a_{jn} = \frac{2}{T}\int_0^T x_j(t) \cos \frac{2n\pi t}{T}\, dt$$

$$b_{jn} = \frac{2}{T}\int_0^T x_j(t) \sin \frac{2n\pi t}{T}\, dt$$

The Fourier coefficients corresponding to the n^{th} harmonic a_{jn} and b_{jn} ($b_{j0} \equiv 0$) are easily found because $x_j(t)$ is piecewise linear and continuous for all time. The derivation of the coefficients here involves the time derivative $\dot{x}_j(t)$, which consists of the sequence of piecewise constant derivatives $\Delta x_{pj}/\Delta t_p$ for values of p in the range of $1 \leq p \leq K$. The time derivative is also periodic with period T and by using this observation it is easily concluded by Kuhl and Giardina[3] that the Fourier coefficients A_{j0}, a_{jn} and b_{jn} are as follows:

$$A_{j0} = \frac{1}{T}\sum_{p=1}^{K} \frac{\Delta x_{pj}}{2\Delta t_p}(t_p^2 - t_{p-1}^2) + \xi_p(t_p - t_{p-1}) \tag{7}$$

where,

$$\xi_p = \sum_{i=1}^{p-1} \Delta x_{pj} - \frac{\Delta x_{pj}}{\Delta t_p} \Delta t_i$$

and,

$$a_{jn} = \frac{T}{2n^2\pi^2}\sum_{p=1}^{K} \frac{\Delta x_{pj}}{\Delta t_p}\left[\cos \frac{2n\pi t_p}{T} - \cos \frac{2n\pi t_{p-1}}{T}\right] \tag{8}$$

$$b_{jn} = \frac{T}{2n^2\pi^2}\sum_{p=1}^{K} \frac{\Delta x_{pj}}{\Delta t_p}\left[\sin \frac{2n\pi t_p}{T} - \sin \frac{2n\pi t_{p-1}}{T}\right] \tag{9}$$

It is useful to be able to specify the number of harmonics M required such that a truncated Fourier approximation to the projection x_j of a closed vector chain have a maximum absolute error no greater than ϵ_j. Let

$$x_{JM} = A_{j0} + \sum_{n=1}^{M} a_{jn} \cos \frac{2n\pi t}{T} + b_{jn} \sin \frac{2n\pi t}{T} \tag{10}$$

It is shown by Giardina and Kuhl[4] that ϵ_j is bounded by the expression

$$\epsilon_j \leq \frac{T}{2\pi^2 M} \cdot \overset{T}{\underset{0}{V}} (\dot{x}_j(t)) \tag{11}$$

For a closed vector chain

$$\overset{T}{\underset{0}{V}} (\dot{x}_j(t)) = \sum_{p=2}^{K} \left|\dot{x}_{pj} - \dot{x}_{p-1j}\right| + \left|\dot{x}_{1j} - \dot{x}_{kj}\right| \tag{12}$$

where,

$$\dot{x}_{pj} = \frac{\Delta x_{pj}}{\Delta t_p}$$

The error of the Fourier approximation to the closed chain code truncated after M harmonics is then bounded by the maximum ϵ_j.

Harmonic descriptions derived from chain codes for 2-dimensional closed figures can be normalized in straight-forward fashion to be independent of size, translation, rotation and dilation and the particular set of descriptors of Wallace and Wintz[5] are used in a later section of this paper to obtain experimental results for aircraft recognition. Another such set of descriptors was developed by Kuhl and Giardina[3]. It is envisioned that similar harmonic descriptors could be developed for higher dimensional closed curves by using the elliptic properties of the Fourier coefficients described by Kuhl.[6]

6. FOURIER DESCRIPTION OF THREE-DIMENSIONAL OBJECTS

This section concerns the identification of three-dimensional rigid objects when viewed by a two-dimensional video imaging system from an arbitrary aspect angle.

If a rigid object in free-space is imaged using visible or somewhat longer wavelengths the resulting image can be digitized, the shape (usually silhouette or contour) extracted, and the object recognized. Two common features used are two-dimensional measurements of the silhouette or contour[7-10] and Fourier descriptors of the contour.[3,5,11-13]

The method used in the experiments described in the later section is Fourier descriptors as discussed in Sec. 5 of this paper fully described in References 5 and 13. These features can be normalized to remove effects of object scale, translation, and rotation about the imaging axis (θ_z in Fig. 3). However two degrees of freedom still remain for movement of the three-dimensional object which correspond to change of viewing aspect angles (θ_x and θ_y in Fig. 3). For an arbitrary three-dimensional object, viewing aspect angle changes will create different two-dimensional representations at the sensor and each view must be stored as a separate library entry for classification purposes. As an example, Fig. 4 shows 77 views of an airplane representing various aspect angles. Those libraries may be stored using the N-dimensional chain code concepts described earlier.

The normalized Fourier descriptor obtained consists of coefficients representing harmonics of the contour. Usually at least 16 harmonics are required (64 real numbers) to

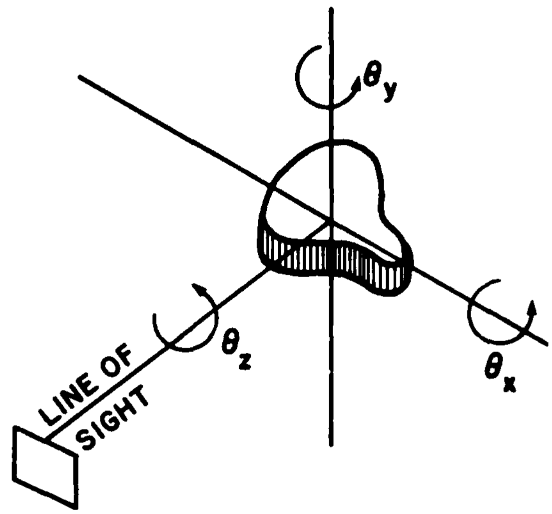

Fig. 3. Coordinate system for imaging three-dimensional object.

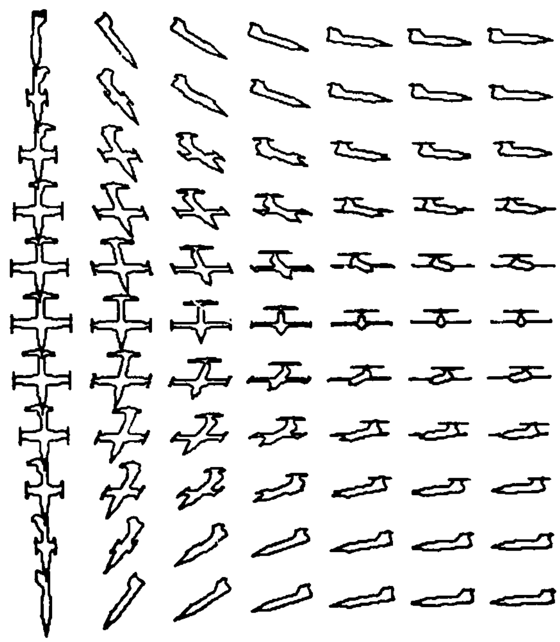

Fig. 4. 2-D views resulting from the aspect angles used for storing the library of an F105 airplane. Horizontally are θ_x angles ranging from 0 to $\pi/2$ radians, and vertically are θ_y angles ranging from $-\pi/2$ to $+\pi/2$ radians.

adequately represent a complex contour. However for a given class of problems such as airplane recognition, considerable correlation exists among these coefficients. Thus an appropriate linear transform (based on eigenvectors of the autocorrelation matrix of all possible NFDs) is used to reduce the dimensionality of the data. Approximately 12 real numbers must be retained to recognize various airplanes from different aspect angles.

Thus if a rigid object in free space is imaged and the resulting contour converted to a normalized (N-dimensional) Fourier Descriptor, the effects of θ_z rotations are normalized out, but changes in θ_x and θ_y give other aspect views which give different N-dimensional vectors.

7. DIFFERENTIAL LIBRARY REPRESENTATION

In order to reduce storage requirements and to speed the classification process, a new method of storing the library information has been investigated. We are considering the library of each object as a set of sampled points from an N-dimensional surface (N represents the number of features retained, 12 in our examples). This surface represents all possible N-dimensional vectors obtained as the aspect angle is changed. It has the topology of a toroid. If we code the N-dimensional vectors independently, we require k·N bytes for each vector where k is the number of bytes used to code each feature value.

Instead, we propose to arrange the sample vectors in some order and code the N-dimensional difference between successive samples. If the difference in each dimension is only allowed to have values of -S, 0, or +S we have a form of N-dimensional delta modulation with step size S. The library can be stored using the N-dimensional code described in Section III.

Three potential advantages are possible using such a scheme:

(1) if the vectors are arranged in an order so that successive vectors are similar (correlated) the differential coding scheme may allow representation of the N-dimensional surface using less storage;
(2) if the distance measure can be incrementally calculated using the differential vector information. As the vectors are sequentially compared, the time required to search the library may be reduced;
(3) when the step size S is small, the intermediate vectors generated between library samples provides a form of interpolation which is much faster than typical interpolation methods.

The potential disadvantages of such a method are:

(1) the quantization error introduced by a fixed step size may reduce classification accuracy;
(2) the storage, speed, and accuracy are highly dependent on vector ordering and step size selection.

Description of Method

The procedure to store the library can be described as follows:

(1) Arrange the N-dimensional vectors in an order which minimizes incremented distances between the vectors (a possible ordering would be by rows as shown in Fig. 5).
(2) Code the first N-dimensional vector directly. Choose a step size S for the differential coding. Replace each subsequent vector by its quantized difference from the previous quantized vector. Each difference vector dimension is quantized by rounding to an integer multiple of the step size S.
(3) Replace each quantized N-dimensional increment vector by a succession of elementary vectors (N-dimensional chain code) with values +1, -1, or 0 in each dimension representing an increase of size S, decrease of size S, or no change in value, respectively.

If aspect angle information is also stored, the angle associated with each incremental vector is linearly interpolated between library vectors.

As an example consider the 3-dimensional increment vector (5,3,-2) with a step size of 1. Assume the aspect angle associated with the previous vector is $\theta_x = 0.5$, $\theta_y = 0.5$ and with the present vector is $\theta_x = 0.0$, $\theta_y = 1.0$. The largest difference value is 5, therefore five elementary vectors will be required to represent this difference vector. Table 1 shows the incremental vector decomposition.

The classification procedure is done by scanning through the library looking for the nearest neighbor in N-dimensional space to the vector representing the normalized unknown object. A faster classification may be obtained by computing each distance by updating the previous distance using the incremental vectors.

The step size selection has a critical effect on performance of this storage procedure. As the step size is increased the following effects are observed:

(1) the number of incremental vectors required decreases and thus the storage required and classification time decreases.
(2) the quantization error is representing the library increase and thus the classification accuracy and aspect angle estimation accuracy decreases.

Table 1. Decomposition into elementary vectors.

Step	Sampled Increment Vector			Chain Code Vector			Angle	
	x1	x2	x3	x1	x2	x3	θ_x	θ_y
1st	1	0.6	-0.4	1	1	0	0.4	0.6
2nd	2	1.2	-0.8	1	0	-1	0.3	0.7
3rd	3	1.8	-1.2	1	1	0	0.2	0.8
4th	4	2.4	-1.6	1	0	-1	0.1	0.9
5th	5	3.0	-2.0	1	1	0	0.0	1.0

Thus the step size selection allows a trade-off between accuracy and the required storage and processing time.

Library Vector Ordering

Another important factor which affects storage, speed, and accuracy is the order in which the library vectors are arranged. The incremental method chosen for storing the library can be thought of as representing the toroidal surface in N-dimensional space with a line in N-dimensional space which traces over the surface. The library vector ordering determines the path that line will take as it attempts to cover the surface.

As an example consider ordering the vectors in row order as shown in Fig. 5. The incremental vector locations when a step size of 350 is used map into the locations shown in Fig 6. Note that small changes in θ_x angle can cause large changes in the N-dimensional vector as evidenced by the large number of incremental vectors at the left center and right center of Fig. 6. If column ordering were used instead of row ordering, the angles asso-

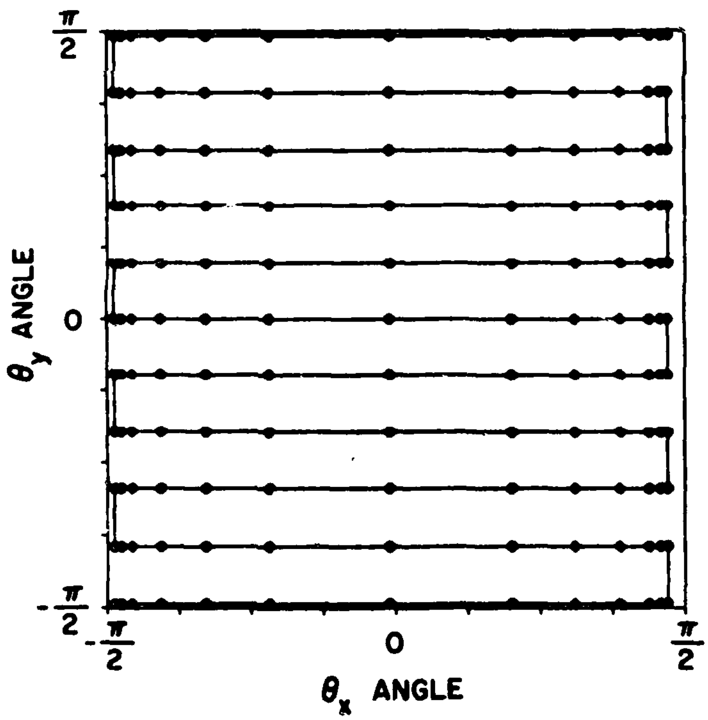

Fig. 5. Library views in row order. Each circle represents an aspect angle used in the original library.

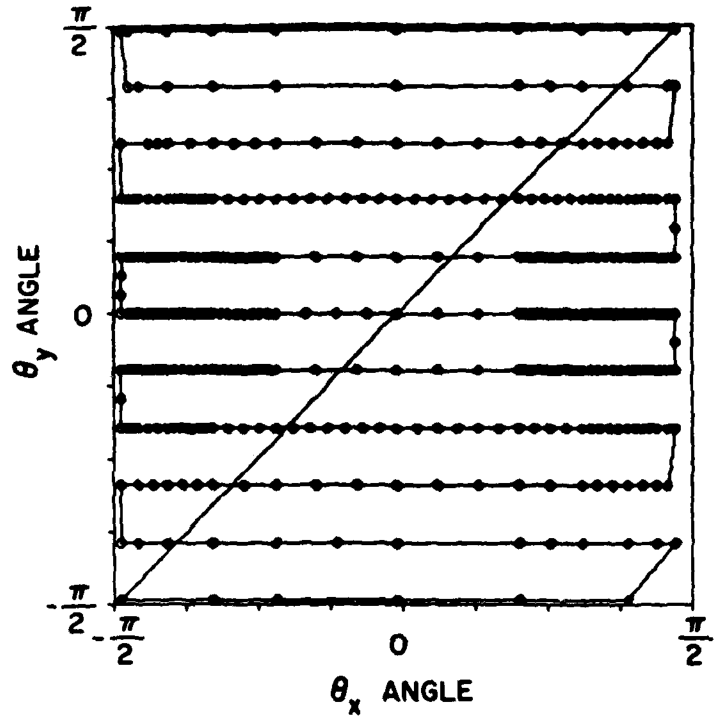

Fig. 6. Library views in row order after chain code representation. Each circle represents the aspect angle for each chain code increment. (Step = 350).

ciated with the incremental vector locations are as shown in Fig. 7.

One might attempt to find the optimal ordering of the library vectors so that the fewest total incremental vectors are required. The problem is to find the shortest path through the library, including each vector once and only once. This is the well known problem of the Traveling Salesman. Finding the optimum solution is feasible only for problems with a small number of nodes. However many heuristic procedures have been developed for approximating the optimal solution. B. Golden et al.[14] have compared some approximate solution algorithms and published an interesting review of this problem. Some are very simple, computational efficient, and give excellent approximation.

The approximation we will use is called "nearest insertion." The procedure is as follows:

(1) Consider all N-dimensional vectors to be nodes.
(2) Start with a subgraph of node i only.
(3) Find node k such that the distance from node i to node k is minimal and form subtour i-k-i.
(4) Find another node k not in the subtree closest to any node in the subtour.
(5) Find the arc (i,j) in the subtour which minimized $d_{i,k} + d_{k,j} - d_{i,j}$ and insert k between i and j.
(6) Go to step (4) unless all nodes are already contained in the path.

It can be shown that the worst case path length using this procedure is less than or equal to twice the optimal length. The algorithm requires on the order of N^2 computations. In our implements each node is selected as a starting node for the procedure and the best result retained. Therefore, the computational complexity becomes $O(N^3)$.

8. Results

As a test of the methods proposed in Section 7 an experiment was run using 6 airplane libraries (mirage, mig, phantom, B-57, F104, and F105). These are the same shapes used in

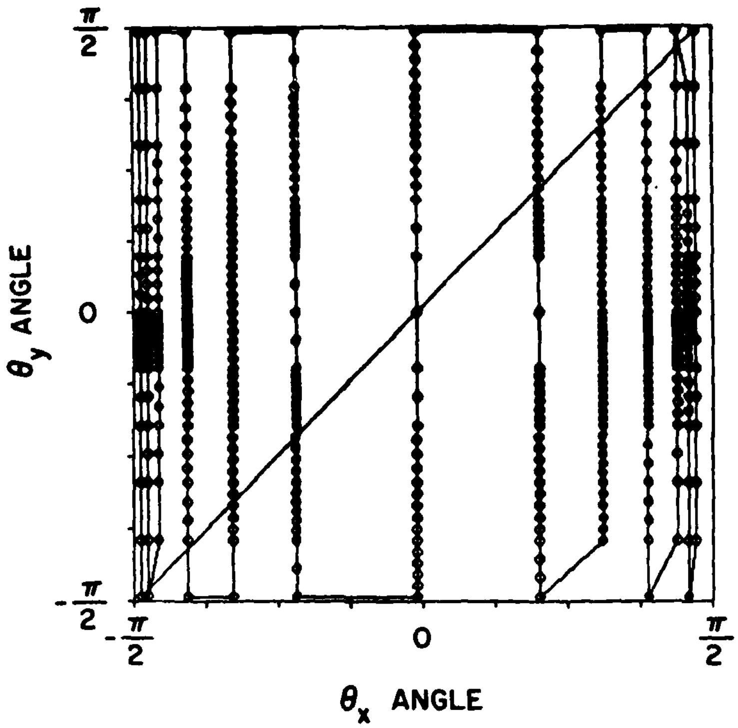

Fig. 7. Library views in column order after chain code representation. Each circle represents the aspect angle for each chain code increment (Step = 350).

References 5, 10, and 13. For all classification experiments, we considered an unknown set composed of 600 views (each are a 128 x 128 grid), 100 for each type of airplane. The normalized Fourier descriptor was calculated and compared to the library entries which were stored in the N-dimensional chain code format described in Section 6.

Shown in Table 2 are some experimental results. For example in entry (1) of Table 2 row ordering of each original library (see Fig. 5) was used prior to converting to a 12-dimensional code. The unknown airplanes were classified correctly with an accuracy of 91%. The median estimated angle error was .075 radians in θ_x (along the rows) and .082 radians in θ_y (along the columns). The storage required for the 12-dimensional chain codes for the 6 libraries was 43.0 k-bytes.

As the step size of the chain code is increased, the quantization error in the library vectors increases, but the storage required decreases. Thus the classification accuracy is decreased but the storage requirements and the processing time decreases. This is shown

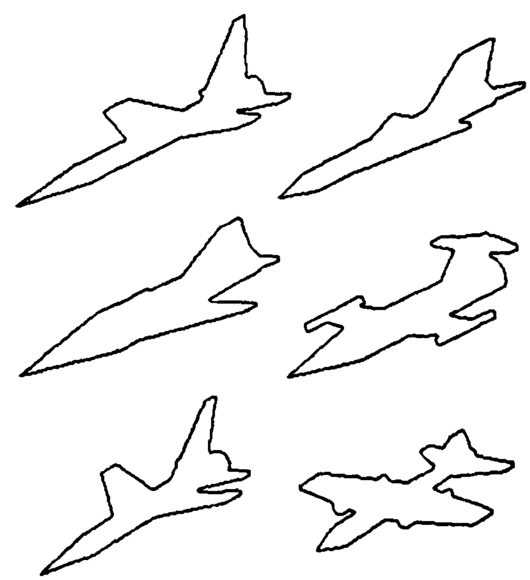

Fig. 8. Representative contours of the six airplanes used for the experimental results.

Table 2 Experimental Results.
Unless otherwise stated: row ordering, absolute distance criterion, N = 12 dimensions, each original library has 143 views.

	Experimental Test (S = step size)	Classification Accuracy (%)	Median Angle Error (radians) θ_x	θ_y	Storage Required (k-bytes)
(1)	S = 100	91.0	.075	.082	43.0
(2)	S = 200	89.8	.080	.082	21.9
(3)	S = 300	88.8	.095	.082	14.7
(4)	S = 400	84.3	.104	.083	11.3
(5)	S = 500	81.5	.109	.083	9.1
(6)	column ordering, S = 200	89.0	.131	.048	28.9
(7)	column ordering, S = 500	81.2	.144	.066	12.0
(8)	optimal ordering, S = 200	89.0	.133	.078	14.4
(9)	optimal ordering, S = 500	82.7	.144	.084	6.4
(10)	mse distance, S = 200	86.8	.071	.082	21.9
(11)	augmented lib (525 views), S = 200	95.8	.051	.046	49.4
(12)	aug. lib, N = 16, S = 200	97.3	.048	.046	60.0

in entries (2) - (5) of Table 2.

If column ordering of the original library is used (see Fig. 6) the accuracy and storage requirements both degrade compared to row ordering (see entries (6) and (7) in Table 2). Thus it is concluded that row ordering results in more average correlation of the sequential 12-dimensional vectors than does column ordering.

Shown in entries (8) and (9) of Table 2 are the results of using the "nearest insertion" vector ordering described at the end of Section 7. Although the storage requirements are better, the accuracy decreases somewhat. Upon close inspection of the details of this ordering, we have found that the views of the airplane for quite different angles may be very similar. The optimal ordering thus tends to have large jumps in aspect angle which results in poor interpolation and degrades classification. Improvements are now underway to limit the optimal ordering to selected regions of aspect angles and we feel classification and angle accuracies will improve significantly.

Entry (10) shows the results of using a mean-square error distance criterion instead of mean absolute distance. Accuracy is somewhat degraded. We feel the mean-square criterion tends to weight the large power low harmonic terms too much while the airplane type is discriminated using the lower power higher harmonic terms.

If the original library is increased to 25 x 21 instead of 13 x 11, the storage required increases as shown in entry (11) but the classification accuracy increases dramatically. Further increase in classification accuracy can be obtained by retaining 16 numbers for each library entry instead of 12. This is shown in entry (12) of Table 2.

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